International Workshop on Functional Analysis on the occasion of the 60th birthday of Carmen Fernández and Antonio Galbis

Burjassot, 19–21 October 2022

Welcome

Carmen Fernández and Antonio Galbis are professors of Mathematical Analysis in the Faculty of Mathematics of the Universitat de València for more than 30 years ago. They investigate in different areas of analysis, like functional analysis, partial differential equations, operator theory, time-frequency analysis and its applications to biomedical signals. In this workshop we celebrate their 60th birthday.

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Global wave front sets and time-frequency analysis

Vicente Asensio

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In this talk, we introduce global wave front sets, using some time-frequency analysis tools such as Gabor frames or the short-time Fourier transform, in global classes of ultradifferentiable functions of Beurling type modulated with weight functions in the sense of Braun, Meise, and Taylor. We compare them with others in the literature, providing sufficient conditions for the equality of them. Finally, we compute the wave front set of some concrete distributions.

Uniformly positive entropy of induced transformations

Nilson C. Bernardes Jr.

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Let (X, T) be a topological dynamical system consisting of a compact metric space X and a continuous onto map $T : X \to X$. Consider the extensions

$$(\mathcal{K}(X), \overline{T}) \quad and \quad (\mathcal{M}(X), \widetilde{T}),$$

where $\mathcal{K}(X)$ is the hyperspace of all nonempty closed subsets of X endowed with the Hausdorff metric, $\mathcal{M}(X)$ is the space of all Borel probability measures on X endowed with the Prohorov metric,

$$\overline{T}(K) = T(K) \quad for \ K \in \mathcal{K}(X), \ and$$

$$(\widetilde{T}(\mu))(B) = \mu(T^{-1}(B)) \quad for \ \mu \in \mathcal{M}(X) \ and \ B \in \mathcal{B}_X$$

($\mathcal{B}_X$ is the Borel σ-algebra of X). The study of the relationships between the dynamics of the systems (X, T), $(\mathcal{K}(X), \overline{T})$ and $(\mathcal{M}(X), \widetilde{T})$ has already become a classic topic in the area of *dynamical systems*. A deep and surprising result due to E. Glasner and B. Weiss (J. Amer. Math. Soc. **8** (1995), 665–686) asserts that:

If (X, T) has zero topological entropy, then so does $(\mathcal{M}(X), \widetilde{T})$.

This result contrasts sharply with the case of the extension to the hyperspace, since there are examples of systems with zero topological entropy whose extensions to the hyperspace have infinite topological entropy. A version to the Glasner-Weiss Theorem to the so-called *null systems* was obtained by D. Kerr and H. Li (Invent. Math. **162** (2005), 649–686):

If (X,T) is a null system, then so is $(\mathcal{M}(X),\widetilde{T})$.

Another important variation of the notion of topological entropy is that of *uniformly positive entropy*, which was introduced by F. Blanchard in 1992 as an analogue in *topological dynamics* for the notion of a K-process in *ergodic thoery*. W. Huang and X. Ye (Israel J. Math. **151** (2006), 237–279) proved that a system (X,T) has uniformly positive entropy if and only if so does its extension to the hyperspace $(\mathcal{K}(X),\overline{T})$. Nevertheless, the case of the extension to the space of probability measures remained open. In a recent paper (Ergodic Theory Dynam. Systems **42** (2022), 9–18) we solved this problem by establishing the following version of the Glasner-Weiss Theorem for the notion of uniformly positive entropy:

(X,T) has uniformly positive entropy if and only if so does $(\mathcal{M}(X),\widetilde{T})$.

Our proof applies techniques from *local entropy theory*. In our talk we will present an overview of this subject. Joint work with Udayan B. Darji and Rômulo M. Vermersch.

Cèsaro-type operators on Hardy spaces

Óscar Blasco

Universitat de València

Given a complex Borel measure $\eta \in M([0,1))$, we study the boundedness of the operator Cèsaro-type operator $\mathcal{C}_\eta$ given by $\mathcal{C}_\eta(f)(z) = \sum_{n=0}^{\infty}(\int_0^1 t^n d\eta(t))(\sum_{k=0}^{\infty} a_k)z^n$, where $f(z) = \sum_{n=0}^{\infty} a_n z^n$, acting on Hardy spaces, BMOA and Bloch. We recover the recent results achieved for positive measures in [1]. We also solve the question that was left open in that paper and show that $C_\mu(H^\infty(\mathbb{D})) \subset$ BMOA whenever μ is a positive Carleson measure on $[0,1)$.

References

[1]   Ganalopoulos, P., Girela, D., Merchan, N. *Cèsaro-like operators acting on spaces of analytic functions,* Anal. Math. Phys. **12** 51 (2022)

Fourier quasicrystals and time-frequency transforms

Paolo Boggiatto

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Fourier quasicrystals are tempered distributions μ which satisfy symmetric conditions on μ and $\widehat{\mu}$. This suggests that techniques from time-frequency analysis could possibly be useful tools in the study of their structures. Recent results in this direction are presented in this talk where we consider "quasicrystals type" conditions on time-frequency representations instead of separately on the distribution and its Fourier transform. More precisely we show that a tempered distribution μ on $\mathbb{R}^d$ whose Wigner transform, $W(\mu)$, is supported on a product of two uniformly discrete sets in $\mathbb{R}^d$ is a quasicrystal. Furthermore, we present some extensions of this result to matrix-Wigner transforms, which include most of the commonly used time-frequency representations.

On embeddings in $C(K)$ spaces having unique Hahn–Banach extensions

Christian Cobollo

Universitat Politècnica de València

In its seminal work of 1960, R. R. Phelps said that a subspace X of a Banach space Y has property U in Y whenever every linear continuous functional defined on X admits a unique norm preserving extension (so called Hahn–Banach extension) to the whole space Y. He initiated a systematic study of the pairs (X, Y) for which X $(\subset Y)$ has property U in Y.

In this talk we are interested in the case in which $Y = C(K)$ for some compact Hausdorff space K, motivated by the classical fact that indeed any Banach space X can be considered as a linear closed subspace of a $C(K)$ space for some K.

Throughout this work, we introduce the concept of U-embedding to denote those linear isometries whose range has property U, and study the existence of such an operator between X and $C(K)$. We provide results when X is finite dimensional, separable, or X also a $C(K)$-space (which relates our work with the concept of linear extension operator introduced in a classical paper by A. Pełczyński).

This is a joint work with Antonio José Guirao and Vicente Montesinos.

Semigroups of composition operators on Hardy spaces of Dirichlet series

Manuel D. Contreras

Universidad de Sevilla

Let $\mathbb{C}_+ := \{w \in \mathbb{C} : \operatorname{Re} w > 0\}$ be the right half-plane. We consider continuous semigroups of holomorphic functions $\{\Phi_t\}_{t \geq 0}$ in the so-called *Gordon-Hedenmalm class* $\mathcal{G}$, that is, the family of holomorphic functions $\Phi : \mathbb{C}_+ \to \mathbb{C}_+$ giving rise to bounded composition operators in the Hardy space of Dirichlet series $\mathcal{H}^2$. We show that there is a one-to-one correspondence between continuous semigroups $\{\Phi_t\}_{t \geq 0}$ in the class $\mathcal{G}$ and strongly continuous semigroups of composition operators $\{T_t\}_{t \geq 0}$, where $T_t(f) = f \circ \Phi_t$, $f \in \mathcal{H}^2$. To provide non-trivial exmaples, we characterize the infinitesimal generators of continuous semigroups in the class $\mathcal{G}$ as those Dirichlet series sending $\mathbb{C}_+$ into its closure.

In the second part of the talk, we extend these results for the range $p \in [1, \infty)$. For the case $p = \infty$, we prove that there is no non-trivial strongly continuous semigroup of composition operators in $\mathcal{H}^\infty$. Finally, we analyze strongly continuous semigroups of composition operators in the setting of the algebra of Dirichlet series.

This is a joint work with Carlos Gómez–Cabello and Luis Rodríguez–Piazza.

Zero-one law of orbital limit points

Karl Grosse–Erdmann

Université de Mons

In 2012, Kit Chan and Irina Seceleanu obtained a remarkable result on the dynamics of unilateral (or bilateral) weighted backward shift operators on $\ell^p(\mathbb{N})$ (or $\ell^p(\mathbb{Z})$): if some orbit under the action of the shift operator has a non-zero limit point then the operator admits a dense orbit, which means that it is hypercyclic. In the bilateral case, their proof is constructive and technically demanding. A new, more abstract, proof allows to extend the result to arbitrary Fréchet sequence spaces in which the natural unit sequences form an unconditional basis. There is also a version for chaotic behaviour.

The talk is based on joint work with Antonio Bonilla.

Frequently recurrent (adjoint) operators

Antoni López–Martínez

Universitat Politècnica de València

We study different pointwise recurrence notions for linear dynamical systems from the Ergodic Theory point of view. We show that from any reiteratively recurrent vector x_0, for an adjoint operator T on a separable dual Banach space X, one can construct a T-invariant probability measure which contains x_0 in its support. This allows us to establish, for these operators, the (surprising) equivalence between reiterative recurrence and frequent recurrence, together with the existence of an invariant measure with full support. Those facts are easily generalized to product and inverse dynamical systems.

References

[1] A. Bonilla, K-G. Grosse-Erdmann, A. López-Martínez and A. Peris: Frequently recurrent operators, *J. Funct. Anal.*, **283**, Issue 12, 2022.

[2] G. Costakis, A. Manoussos and I. Parissis: Recurrent linear operators, *Complex Anal. Oper. Theory* **8** (2014), 1601–1643.

[3] H. Furstenberg: *Recurrence in Ergodic Theory and Combinatorial Number Theory*. Princeton University Press, New Jersey 1981.

[4] S. Grivaux and A. López-Martínez: Recurrence properties for linear dynamical systems: an approach via invariant measures, *J. Math. Pures Appl.*, (accepted) 2022, to appear.

[5] S. Grivaux and E. Matheron: Invariant measures for frequently hypercyclic operators, *Adv. Math.*, **265** (2014), p. 371–427.

Norm attaining functions on $H_\infty(B_\mathbb{C}^N)$ and $\mathcal{H}_\infty(\mathbb{C}_+)$

Manolo Maestre

Universidad de Valencia

This is preliminary report on work with Richard Aron and Pepe Bonet.

The main aim of this talk is the following. Consider $H^\infty(\mathbb{D})$ the Banach algebra of bounded holomorphic functions on the complex unit disk $\mathbb{D}$ endowed with the supremum norm $\|\cdot\|_\infty$. By the maximum modulus principle the only functions in $H^\infty(\mathbb{D})$ that attain their norm are the constant functions. But it is known that $H^\infty(\mathbb{D})$ is a dual space. Hence, by the Bishop–Phelps theorem, the set of elements of $H^\infty(\mathbb{D})$ that attain their norm with respect to that (unique) predual Banach space is a dense set. We will present a characterization of elements of $H^\infty(\mathbb{D})$ which are norm attaining and study the situation for $H^\infty(\mathbb{D}^N)$ and $H^\infty(B_N)$, were B_N is the N-euclidean norm.

Also we study similar situation when we consider $\mathcal{H}_\infty(\mathbb{C}_+)$ the Banach algebra of the Dirichlet series convergent and bounded on $\mathbb{C}_+$, the right half plane.

Associated weights for spaces of p-integrable entire functions

Elisabetta Mangino

Università del Salento

In analogy to the notion of associated weights for weighted spaces of analytic functions with sup-norms, p-associated weights are introduced for spaces of entire p-integrable functions, $1 \le p < \infty$. As an application, necessary conditions for the boundedness of composition operators acting between general Fock type spaces are proved.

Diagonal operators between sequence (LF)-spaces

Claudio Mele

Università del Salento

Diagonal (multiplication) operators acting between a particular class of countable inductive spectra of Fréchet sequence spaces, called sequence (LF)-spaces, are investigated. We prove results concerning boundedness, compactness, power boundedness and mean ergodicity. Furthermore, we determine when a diagonal operator is Montel and reflexive. A completely characterization of the spectrum in terms of the system of weights defining the spaces is analyzed too.

References

[1] J. Bonet, E. Jordá, A. Rodríguez-Arenas, *Mean ergodic multiplication operators on weighted spaces of continuous functions*, Mediterr. J. Math. **15**, 108 (2018). https://doi.org/10.1007/s00009-018-1150-8.

[2] C. Mele, *On diagonal operators between the sequence (LF)-spaces $l_p(\mathcal{V})$*, Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat. **116**, 141 (2022). https://doi.org/10.1007/s13398-022-01284-8

Common hypercyclicity and covering

Quentin Menet

Université de Mons

Given a family of operators defined on the same Banach space, we would like to determine whether there exists a vector whose orbit is dense under the action of each of these operators. If the considered family is countable, it is well known that such a vector exists if and only if each operator admits a dense orbit. However, when the family is non-countable, the situation becomes more complicated and we need to find a countable covering of the set of parameters underlying our family of operators. In this talk we will try to understand when such a vector and thus when such a covering exists for a non-countable family of products of multiples of the backward shift.

Estimates between time-frequency representations in weighted spaces and Donoho–Stark type uncertainty principles

Alessandro Oliaro

Università degli studi di Torino

In this talk we give different estimates between Lebesgue norms of quadratic time-frequency representations. We show that in some cases it is not possible to have such bounds in classical L^p spaces, but the Lebesgue norm needs to be suitably weighted. This leads to consider weights of polynomial type, and more generally of ultradifferentiable type, and this in turn gives rise to use as functional setting the ultradifferentiable classes. From such estimates we deduce uncertainty principles of Donoho–Stark type saying, roughly speaking, that if the content of a time-frequency representation in a Lebesgue measurable set $E \subset \mathbb{R}^{2N}$ is greater than (a fraction of) the whole content of another time-frequency representation, then the set E must have sufficiently large measure. The talk is based on a work in collaboration with Angela Albanese and Claudio Mele (University of Salento).

Time-frequency analysis for biomedical signal processing. Applications to electrocardiogram signals

Nuria Ortigosa

Universitat Politècnica de València

In the last years, time-frequency analysis has arised as a relevant tool for biomedical signal processing. Analysis in the frequency domain is a well standardized tool for characterization of many clinical and physiological phenomena. However, biomedical signals are characterized by a dynamical evolution, so that time-frequency analysis can overcome the main limitation of non-stationarity. In this talk principal methods of time-frequency analysis will be briefly described, as well as examples of successful application to electrocardiogram signals will be presented: atrial fibrillation characterization and classification, and early prediction of response to cardiac resynchronization therapy.

Hyperspace and fuzzy dynamics for operators on locally convex spaces

Alfred Peris

Universitat Politècnica de València

Since the 1975 paper of Bauer and Sigmund [1], the study of hyperspace dynamics induced by a continuous map $f : X \to X$ on a topological space X has experimented a great development, especially for those notions related to wild behaviour. In the case of hyperspace dynamics induced by an operator $T : X \to X$ on a Banach space X, the first study was given by Herzog and Lemmert [3]. Strongly related to the hyperspace dynamics is the fuzzy dynamics induced by f.

Our intention is to review on dynamical properties that are inherited on the hyperspace and fuzzy spaces when induced by operators on locally convex spaces. More precisely, we will talk about the following properties:

1. Devaney chaos.

2. Li-Yorke chaos.

3. $\mathcal{F}$-transitivity for a Furstenberg family $\mathcal{F}$.

References

[1] Bauer, W., Sigmund, K., Topological dynamics of transformations induced on the space of probability measures. (1975) Monatshefte für Mathematik, 79 (2), pp. 81-92.

[2] Bernardes, N.C., Peris, A., Rodenas, F., Set-Valued Chaos in Linear Dynamics. (2017) Integral Equations and Operator Theory, 88 (4), pp. 451-463.

[3] Herzog, G., Lemmert, R., On universal subsets of Banach spaces. (1998) Math. Z., 229 (4), pp. 615-619.

[4] Martínez-Giménez, F., Peris, A., Rodenas, F., Chaos on fuzzy dynamical systems. (2021) Mathematics, 9 (20), art. no. 2629.

[5] Peris, A. Set-valued discrete chaos. (2005) Chaos, Solitons and Fractals, 26 (1), pp. 19-23.

Optimal flat funtions and right inverses for the asymptotic Borel map in ultraholomorphic classes

Javier Sanz

Universidad de Valladolid

20 Oct
09:50

We prove the existence of optimal flat functions in Carleman–Roumieu ultraholomorphic classes, defined by general weight sequences, in sectors in suitably restricted opening. The key fact is the interpretation of a condition of M. Langenbruch, recently recovered by D. N. Nenning, A. Rainer and G. Schindl in a mixed setting, in terms of a property of regular variation.

If the defining sequence is regular in the sense of Dyn'kin, from these optimal flat functions one may obtain local right inverses for the asymptotic Borel map, that interpolate in Banach spaces of asymptotic power series with a control of the type. Finally, the existence of global right inverses between the corresponding (LB) spaces is investigated, by extending previous results by J. Schmets and M. Valdivia and by A. Debrouwere. Again the theory of regular variation sheds some light on the role of condition (β_2) of H. J. Petzsche in this regard.

This is joint work with J. Jiménez–Garrido (University of Cantabria, Spain), I. Miguel (University of Valladolid) and G. Schindl (University of Vienna, Austria).

Compact weighted composition operators on spaces of holomorphic functions

Daniel Santacreu

Universitat Politècnica de València

20 Oct
18:20

Given an infinite dimensional Banach space X and its open unit ball B, we study when the weighted composition operator $C_{\psi,\varphi}$ is compact in the space of all bounded analytic functions $H^\infty(B)$, and when it is bounded, reflexive, Montel and (weakly) compact in the space of analytic functions of bounded type $H_b(B)$. The study is given in terms of properties of the weight ψ and the symbol φ.

Joint work with José Bonet, David Jornet and Pablo Sevilla–Peris.

Analytic pseudo-differential calculus via the Bargmann transform

Joachim Toft

Linnaeus University

The Bargmann transform maps Fourier-invariant function spaces and their duals to spaces of formal power series expansions, which sometimes are convenient classes of analytic functions.

In the 70th, Berezin used the Bargmann transform to translate problems in operator theory into an analytic pseudo-differential calculus, the so-called Wick calculus, where the involved symbols are analytic functions, and the corresponding operators map suitable classes of entire functions into other classes of entire functions. In the same manner, the Toeplitz operators correspond to so-called anti-Wick operators on the Bargmann transformed side.

Recently, the author performed some investigations on the so-called Pilipović spaces, defined by imposing suitable boundaries on the Hermite coefficients of the involved functions or distributions. The family of Pilipović spaces contains all Fourier invariant classical Gelfand–Shilov spaces and other subspaces of such Gelfand–Shilov spaces. In the same way, the family of Pilipović distribution spaces contains spaces which are strictly larger than any Fourier invariant Gelfand–Shilov distribution space.

In the talk we find convenient characterisations of Wick and anti-Wick operators acting on the Bargmann images of Pilipović spaces. We also discuss some links between global ellipticity in the real pseudo-differential calculus and the Wick calculus, as well as links between Wick and anti-Wick operators.

Important prerequisites for the investigations concerns Bargmann images of Pilipović spaces, which were partially obtained by the author in collaboration with C. Fernandez and A. Galbis in [1].

The talk is based on collaborations with *Nenad Teofanov* and *Patrik Wahlberg*. Relevant for the talk is:

[1] C. Fernández, A. Galbis, J. Toft *The Bargmann transform and powers of harmonic oscillator on Gelfand–Shilov subspaces*, RACSAM **111** (2017), 1–13.

[2] N. Teofanov, J. Toft *Pseudo-differential calculus in a Bargmann setting*, Ann. Acad. Sci. Fenn. Math. **45** (2020), 227–257.

[3] N. Teofanov, J. Toft, P. Wahlberg *Pseudo-differential calculus in a Bargmann setting*, J. Math. Pures Appl., appeared online 2022.

[4] J. Toft *Images of function and distribution spaces under the Bargmann transform*, J. Pseudo-Differ. Oper. Appl. **8** (2017), 83–139.

[5] J. Toft *Wick and Anti-Wick Characterizations of Linear Operators on Spaces of Power Series Expansions*, J. Four. Anal. Appl. **28** (2022), paper no. 71.