Local classes of operators which satisfy a Bollobás type theorem.

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A joint work with
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V Congreso de Jóvenes Investigadores de la RSME.
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- $T \in \mathrm{NRA}(X)$ if $\exists (x, x^*) \in \Pi(X)$ such that $|\langle x^*, T(x)\rangle| = \nu(T)$.

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A pair of Banach spaces (X, Y) is said to have the **BPBp** if for every $\varepsilon \in (0, 1)$, there exists $\eta(\varepsilon) > 0$ such that if $T \in S_{\mathcal{L}(X,Y)}$ and $x \in S_X$ satisfy

$$\|T(x)\| > 1 - \eta(\varepsilon),$$

then there exist $S \in \mathcal{L}(X, Y)$ with $\|S\| = 1$ and $x_0 \in S_X$ such that

$$\|S(x_0)\| = 1, \quad \|x_0 - x\| < \varepsilon, \quad \text{and} \quad \|T - S\| < \varepsilon.$$

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A Banach space X is said to have the **BPBp-nu** if for every $\varepsilon \in (0, 1)$, there exists $\eta(\varepsilon) > 0$ such that if $T \in \mathcal{L}(X)$ and $(x, x^*) \in \Pi(X)$ satisfy

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then there are $S \in \mathcal{L}(X)$, $(y, y^*) \in \Pi(X)$ with $\nu(S) = |\langle y^*, S(y)\rangle| = 1$ and

$$\|y - x\| < \varepsilon, \quad \|y^* - x^*\| < \varepsilon, \quad \|T - S\| < \varepsilon.$$

A change of point of view

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Space

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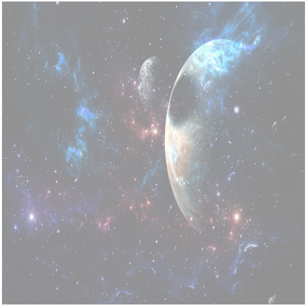

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Operator

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$\mathcal{A}_{\|\cdot\|}(X, Y)$ is the set of all norm attaining operators $T \in S_{\mathcal{L}(X,Y)}$ such that if $\varepsilon > 0$, then there is $\eta(\varepsilon, T) > 0$ such that whenever $x \in S_X$ satisfies $\|T(x)\| > 1 - \eta(\varepsilon, T)$, there exists $x_0 \in S_X$ with

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Definition: the $\mathcal{A}_{\mathrm{nu}}(X)$ class

$\mathcal{A}_{\mathrm{nu}}(X)$ is the set of all numerical radius attaining operators $T \in \mathcal{L}(X)$ with $\nu(T) = 1$ such that if $\varepsilon > 0$, then there is $\eta(\varepsilon, T) > 0$ such that whenever $(x, x^*) \in \Pi(X)$ satisfies $|\langle x^*, T(x)\rangle| > 1 - \eta(\varepsilon, T)$, there is $(x_0, x_0^*) \in \Pi(X)$ with

$$|\langle x_0^*, T(x_0)\rangle| = \nu(T) = 1, \quad \|x_0 - x\| < \varepsilon, \quad \text{and} \quad \|x_0^* - x^*\| < \varepsilon.$$

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- $x^* = \left(1, \frac{1}{2}, \frac{2}{3}, \ldots, \frac{n-1}{n}, \ldots\right) \in \ell_\infty = \ell_1^*$.

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- Viewing ℓ_∞ as a real space, $x^* = \left(\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots\right) \in \ell_1$ embedded in ℓ_∞^*.

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M. Acosta gave in 1990 an example of a non compact operator S on a separable Hilbert space H with $\|S\| = \nu(S) = 1$ such that $S \notin \mathcal{A}_{\mathrm{nu}}(H)$, since $S \notin \mathrm{NRA}(H)$.

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Seeing c_0 as a real space, set $T : c_0 \to c_0$ and $T^* : \ell_1 \to \ell_1$ as follows:

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For a compact operator $T \in \mathcal{K}(H)$ on a Hilbert space H, the condition $\|T\| = \nu(T) = 1$ implies $T \in \mathcal{A}_{\mathrm{nu}}(H)$. Is the converse true?

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- Note that T cannot belong to $\mathcal{A}_{\|\cdot\|}(H, H)$, since $\|T\| > 1$.

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Some more definitions

A new operator

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Connecting the sets $\mathcal{A}_{\|\cdot\|}$ and $\mathcal{A}_{\text{nu}}$ via direct sums

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X	Y	A_no
FinDim	BS	S_L
c0	K	NAS
UnConv BS	K	S_L
l1	K	NAS
linf	K	NAS
lp	lp	NAS∩NRAS
BS	X	Isometries
Ref+KK	BS	S_K
Ref+KK	Schur	S_L
l1	l1	S_K
c0	c0	S_L∩nuS_L
BS	X	A_nu
BS	UnSmooth BS	$T \Rightarrow T^{**}$
UnConv BS	BS	$T^{**} \Rightarrow T$
c0	c0	$T^{**} \not\Rightarrow T$
l1	l1	$T \not\Rightarrow T^{**}$
c0, lp. 1<p<=inf	X	P_N

X	Y	A_no∩A_nu
c0	c0	S, S^{**}

W	Z	⊕	A_no	A_nu
UnSmooth	UnSmooth	1	T	⇒ˆT
l1	l1	1	T	⇏ˆT
BS	BS	Abs type 1	T	⇏ˆT
l2	l2	1	T⇍	ˆT
BS	UnConv+UnSmooth	Inf	T	⇒ˆT
l1	l1	Inf	T	⇏ˆT
BS	BS	Abs type inf	T	⇏ˆT
l2	l2	Inf	T⇍	ˆT

X	A_nu
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lp	NAS∩NRAS
l2	Isometries
Ref+KK+FrDif	S_K∩nuS
BS	Identity
H	S_K∩nuS
Sep H	S_L∩nuS_L
c0	S_L∩nuS_L
l1	NRAS_K∩A_no
Sep InfDim H	NRAS_k\S
Ref	$T \Leftrightarrow T^{**}$
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c0, lp. 1<p<=inf	P_N

Local classes of operators which satisfy a Bollobás type theorem.

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