Weighted composition operators on Korenblum type spaces of analytic functions

Esther Gómez Orts
Universitat Politècnica de València

Work supervised by José Bonet and Pablo Galindo

January 30, 2020

5º Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española

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- Weighted composition operator:
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  - Contreras
  - Cowen
  - Eklund
  - Gunatillake
  - Hernández-Díaz
  - Kamowitz
  - Lindström
  - McCluer
  - Mleczko
  - Montes–Rodríguez
  - Rzeczkowski
  - Shapiro
  - Zhu
  - ...

Precedents

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  - Korenblum
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Weighted Banach spaces

$H(\mathbb{D})$ space of all analytic functions on $\mathbb{D}$, endowed with the τ_{co} topology.

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Definition

For each $\alpha > 0$,

- $H_\alpha^\infty := \left\{ f \in H(\mathbb{D}) \colon \|f\|_\alpha := \sup_{z\in\mathbb{D}} (1-|z|)^\alpha |f(z)| < \infty \right\}$,
- $H_\alpha^0 := \left\{ f \in H(\mathbb{D}) \colon \lim_{|z|\to 1^-} (1-|z|)^\alpha |f(z)| = 0 \right\}$.

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If $0 < \alpha_1 < \alpha_2$, the inclusion $H_{\alpha_1}^\infty \hookrightarrow H_{\alpha_2}^0$ is compact.

Korenblum type spaces

- $A_+^{-\alpha} := \bigcap_{\beta>\alpha} H_\beta^\infty = \bigcap_{\beta>\alpha} H_\beta^0 = \operatorname{proj}_k H_{\alpha+\frac{1}{k}}^\infty = \operatorname{proj}_k H_{\alpha+\frac{1}{k}}^0.$

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- $A_-^{-\alpha} := \bigcup_{\beta<\alpha} H_\beta^\infty = \bigcup_{\beta<\alpha} H_\beta^0 = \operatorname{ind}_k H_{\alpha-\frac{1}{k}}^\infty = \operatorname{ind}_k H_{\alpha-\frac{1}{k}}^0.$

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- $A^{-\infty} := \bigcup_{k\in\mathbb{N}} H_k^\infty = \bigcup_{k\in\mathbb{N}} H_k^0 = \operatorname{ind}_k H_k^\infty = \operatorname{ind}_k H_k^0.$

Weighted composition operator

Definition

Let $\varphi : \mathbb{D} \to \mathbb{D}$ and $\psi : \mathbb{D} \to \mathbb{C}$ be analytic.

$$W_{\psi,\varphi}(f(z)) := \psi(z)f(\varphi(z)), \quad z \in \mathbb{D}.$$

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$W_{\psi,\varphi} = M_\psi \circ C_\varphi.$

Contents

Characterizations of continuity

From the characterization for H_α^∞ of Contreras and Hernández-Díaz (2000) ...

Proposition

- Let $\alpha \geq 0$. $W_{\psi,\varphi} : A_+^{-\alpha} \to A_+^{-\alpha}$ is continuous if and only if $\forall \varepsilon > 0$ $\exists \delta \in]0, \varepsilon]$ such that

$$\sup_{z \in \mathbb{D}} \frac{|\psi(z)|(1-|z|)^{\alpha+\varepsilon}}{(1-|\varphi(z)|)^{\alpha+\delta}} < \infty.$$

  If this is the case, then $\psi \in A_+^{-\alpha}$.

- Let $0 < \alpha \leq \infty$. $W_{\psi,\varphi} : A_-^{-\alpha} \to A_-^{-\alpha}$ is continuous if and only if $\forall \varepsilon > 0$ $\exists \delta \in]0, \varepsilon]$ such that

$$\sup_{z \in \mathbb{D}} \frac{|\psi(z)|(1-|z|)^{\alpha-\delta}}{(1-|\varphi(z)|)^{\alpha-\varepsilon}} < \infty.$$

  If this is the case, then $\psi \in A_-^{-\alpha}$.

Conditions of continuity

Corollary

- For $\alpha \geq 0$, if $\psi \in A_{+}^{-0}$, then $W_{\psi,\varphi} \in \mathcal{L}(A_{+}^{-\alpha})$.
- For $\alpha > 0$, if $\psi \in A_{+}^{-0}$, then $W_{\psi,\varphi} \in \mathcal{L}(A_{-}^{-\alpha})$.

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- For $\alpha > 0$, if $\psi \in A_+^{-0}$, then $W_{\psi,\varphi} \in \mathcal{L}(A_-^{-\alpha})$.

Example

Take $\varphi(z) = z/2$, for all $z \in \mathbb{D}$. For each $\psi \in A_+^{-\alpha} \setminus A_+^{-0}$, $W_{\psi,\varphi}$ is continuous on $A_+^{-\alpha}$. In the characterization take $\delta = \varepsilon$, then

$$\sup_{z\in\mathbb{D}} |\psi(z)| \frac{(1-|z|)^{\alpha+\varepsilon}}{(1-|z|/2)^{\alpha+\varepsilon}} \leq 2^{\alpha+\varepsilon} \sup_{z\in\mathbb{D}} |\psi(z)|(1-|z|)^{\alpha+\varepsilon} < \infty.$$

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Proposition

$W_{\psi,\varphi} \in \mathcal{L}(A^{-\infty})$ if and only if $\psi \in A^{-\infty}$.

Characterizations of compactness

Proposition

- Let $\alpha \geq 0$. $W_{\psi,\varphi} : A_+^{-\alpha} \to A_+^{-\alpha}$ is compact if and only if it is continuous and $\exists \varepsilon > 0$ such that $\forall \delta \in]0, \varepsilon]$

$$\sup_{z\in\mathbb{D}} \frac{|\psi(z)|(1-|z|)^{\alpha+\delta}}{(1-|\varphi(z)|)^{\alpha+\varepsilon}} < \infty.$$

- Let $0 < \alpha \leq \infty$. $W_{\psi,\varphi} : A_-^{-\alpha} \to A_-^{-\alpha}$ is compact if and only if it is continuous and $\exists \varepsilon < 0$ such that $\forall \delta \in [0, \varepsilon[$

$$\sup_{z\in\mathbb{D}} \frac{|\psi(z)|(1-|z|)^{\alpha-\varepsilon}}{(1-|\varphi(z)|)^{\alpha-\delta}} < \infty.$$

Conditions of compactness

Corollary

- Let $\alpha \geq 0$. If $W_{\psi,\varphi} : A_+^{-\alpha} \to A_+^{-\alpha}$ is compact, then $\exists \eta > \alpha$ such that $W_{\psi,\varphi} : H_\eta^0 \to H_\eta^0$ is compact.
- Let $0 < \alpha \leq \infty$. If $W_{\psi,\varphi} : A_-^{-\alpha} \to A_-^{-\alpha}$ is compact, then $\exists \gamma < \alpha$ such that $W_{\psi,\varphi} : H_\gamma^0 \to H_\gamma^0$ is compact.

Conditions of compactness

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- Let $\alpha \geq 0$. If $W_{\psi,\varphi} : A_+^{-\alpha} \to A_+^{-\alpha}$ is compact, then $\exists \eta > \alpha$ such that $W_{\psi,\varphi} : H_\eta^0 \to H_\eta^0$ is compact.
- Let $0 < \alpha \leq \infty$. If $W_{\psi,\varphi} : A_-^{-\alpha} \to A_-^{-\alpha}$ is compact, then $\exists \gamma < \alpha$ such that $W_{\psi,\varphi} : H_\gamma^0 \to H_\gamma^0$ is compact.

Corollary

Assume $\exists\, 0 < r < 1$, such that $|\varphi(z)| \leq r$ for all $z \in \mathbb{D}$. If $W_{\psi,\varphi} : A_+^{-\alpha} \to A_+^{-\alpha}, \alpha \geq 0$, (resp. $W_{\psi,\varphi} : A_-^{-\alpha} \to A_-^{-\alpha}, \alpha > 0$) is continuous, then $W_{\psi,\varphi}$ is compact.

Characterizations of invertibility

From Bourdon (2014) ...

Proposition

- For $\alpha \geq 0$, $W_{\psi,\varphi}$ is invertible on $A_+^{-\alpha}$ if and only if $\varphi \in \mathrm{Aut}(\mathbb{D})$ and $\psi, 1/\psi \in A_+^{-0}$.
- For $\alpha > 0$, $W_{\psi,\varphi}$ is invertible on $A_-^{-\alpha}$ if and only if $\varphi \in \mathrm{Aut}(\mathbb{D})$ and $\psi, 1/\psi \in A_+^{-0}$.
- $W_{\psi,\varphi}$ is invertible on $A^{-\infty}$ if and only if $\varphi \in \mathrm{Aut}(\mathbb{D})$ and $\psi, 1/\psi \in A^{-\infty}$.

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- $W_{\psi,\varphi}$ is invertible on $A^{-\infty}$ if and only if $\varphi \in \mathrm{Aut}(\mathbb{D})$ and $\psi, 1/\psi \in A^{-\infty}$.

Example

Consider $\psi(z) := \log(z+1) - 5$, $z \in \mathbb{D}$.

- $\psi \in A_+^{-0}, A^{-\infty}$,
- $1/\psi \in H^\infty$,
- $\psi \notin H^\infty$.

Contents

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- $\varphi(0) = 0, 0 < |\varphi'(0)| < 1$

Essential norm and radius

Definition

Let X be a Banach space, $T \in \mathcal{L}(X)$. The *essential norm* of T is defined as

$$\|T\|_e := d(T, \mathcal{K}(X)).$$

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Theorem (Montes-Rodríguez)

The continuous weighted composition operators $W_{\psi,\varphi} : H_\alpha^\infty \to H_\alpha^\infty$ and $W_{\psi,\varphi} : H_\alpha^0 \to H_\alpha^0$ satisfy that their essential norm is given by

$$||W_{\psi,\varphi}||_e = \lim_{r\to 1} \sup_{|\varphi(z)|>r} |\psi(z)| \frac{(1-|z|)^\alpha}{(1-|\varphi(z)|)^\alpha} .$$

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The continuous weighted composition operators $W_{\psi,\varphi} : H^\infty_\alpha \to H^\infty_\alpha$ and $W_{\psi,\varphi} : H^0_\alpha \to H^0_\alpha$ satisfy that their essential norm is given by

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Essential spectral radius:

$$r_e(W_{\psi,\varphi}, H^\infty_\alpha) = r_e(W_{\psi,\varphi}, H^0_\alpha) = \lim_n ||W^n_{\psi,\varphi}||_e^{1/n}.$$

Point spectrum of C_φ

From Kamowitz (1979) ...

Proposition

Suppose $W_{\psi,\varphi} : A \to A$ is continuous where $A = A_+^{-\alpha}$, $\alpha \geq 0$ or $A = A_-^{-\alpha}$, $0 < \alpha < \infty$. Then,

$$\{\varphi'(0)^n\}_{n=0}^{\infty} \setminus \overline{B}(0, r_e(C_\varphi, H_\alpha^\infty)) \subseteq \sigma_p(C_\varphi, A) \subseteq \{\varphi'(0)^n\}_{n=0}^{\infty}.$$

Spectrum on $A_+^{-\alpha}$ and $A_-^{-\alpha}$

From Kamowitz (1979) and Aron, Lindström (2004) ...

Theorem

Suppose $W_{\psi,\varphi} : A \to A$ is continuous where $A = A_+^{-\alpha}$, $\alpha \geq 0$ or $A = A_-^{-\alpha}$, $0 < \alpha < \infty$. Then,

$$\{0\} \cup \{\psi(0)\varphi'(0)^n\}_{n=0}^{\infty} \subseteq \sigma(W_{\psi,\varphi}, A) \subseteq \overline{B}(0, L) \cup \{\psi(0)\varphi'(0)^n\}_{n=0}^{\infty},$$

where $L = \lim_{\beta\to\alpha} r_e(W_{\psi,\varphi}, H_\beta^\infty)$.

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where $L = \lim_{\beta\to\alpha} r_e(W_{\psi,\varphi}, H_\beta^\infty)$.

Corollary

If $A = A_+^{-\alpha}$, $\alpha \geq 0$ or $A = A_-^{-\alpha}$, $0 < \alpha < \infty$, then

$$\{0\} \cup \{\varphi'(0)^n\}_{n=0}^{\infty} \subseteq \sigma(C_\varphi, A) \subseteq \overline{B}(0, r_e(C_\varphi, H_\alpha^\infty)) \cup \{\varphi'(0)^n\}_{n=0}^{\infty}.$$

Spectrum on $A^{-\infty}$

Theorem

- $\sigma_p(C_\varphi, A^{-\infty}) = \{\varphi'(0)^n\}_{n=0}^{\infty}$,
- $\sigma(C_\varphi, A^{-\infty}) = \{0\} \cup \{\varphi'(0)^n\}_{n=0}^{\infty}$.

Spectrum and point spectrum of M_ψ

Proposition

If M_ψ is continuous on $A_+^{-\alpha}, \alpha \geq 0$, or $A_-^{-\alpha}, 0 < \alpha \leq \infty$ for some non-constant function $\psi \in H(\mathbb{D})$, then $\sigma_p(M_\psi) = \emptyset$ and $\psi(\mathbb{D}) \subseteq \sigma(M_\psi) \subseteq \overline{\psi(\mathbb{D})}$.

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Example

Take $\psi(z) := \frac{1}{1-z}$, $z \in \mathbb{D}$. M_ψ is continuous on $A^{-\infty}$. Observe $\frac{1}{2} = \psi(-1) \in \overline{\psi(\mathbb{D})}$, but $\frac{1}{2} \in \rho(M_\psi, A^{-\infty})$ because $M_{\frac{1}{\psi - \frac{1}{2}}} \in A^{-\infty}$ and is the inverse.

Spectrum and point spectrum when φ is a rotation

Lemma

Let $\varphi \in H(\mathbb{D})$, $\varphi(z) = cz$, $z \in \mathbb{D}$, with $|c| = 1$. Then

(i) $\sigma_p(C_\varphi, H_\alpha^\infty) = \{c^n\}_{n=0}^\infty$,

(ii) If c is a root of unity, then $\sigma(C_\varphi, H_\alpha^\infty) = \sigma_p(C_\varphi, H_\alpha^\infty) = \{c^n\}_{n=0}^\infty$,

(iii) If c is not a root of unity, then $\sigma(C_\varphi, H_\alpha^\infty) = \partial\mathbb{D}$.

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Theorem

Let $\varphi(z) = cz$, $z \in \mathbb{D}$, with $|c| = 1$. If A is $A_+^{-\alpha}$, $\alpha \geq 0$ or $A_-^{-\alpha}$, $0 < \alpha \leq \infty$, then

(i) $\sigma_p(C_\varphi, A) = \{c^n\}_{n=0}^\infty$,

(ii) if c is a root of unity, $\sigma(C_\varphi, A) = \sigma_p(C_\varphi, A) = \{c^n\}_{n=0}^\infty$,

(iii) if c is not a root of unity, $\{c^n\}_{n=0}^\infty \subseteq \sigma(C_\varphi, A) \subseteq \partial\mathbb{D}$.

Case $c^n \neq 1 \; \forall n \in \mathbb{N}$, in $A^{-\infty}$

Theorem

Let $\varphi(z) = cz, z \in \mathbb{D}, |c| = 1$ and c is not a root of unity. Take $\lambda \neq 1, |\lambda| = 1$. Then, the following are equivalent:

- $\lambda \in \rho(C_\varphi, A^{-\infty})$,
- $\exists\, s \geq 1$ and $\varepsilon > 0$ such that $|c^n - \lambda| \geq \varepsilon n^{-s}$ for each $n \in \mathbb{N}$.

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Theorem (Bonet)

Let $\varphi(z) = cz, z \in \mathbb{D}, |c| = 1$ and c is not a root of unity. Take $\lambda \in \mathbb{C}$ with $|\lambda| = 1$. Then, the following are equivalent:

- $\lambda \in \rho(C_\varphi, H_0(\mathbb{D}))$,
- for each $0 < \varepsilon < 1\ \exists \delta(\varepsilon) > 0$ such that $|c^n - \lambda| \geq \delta(\varepsilon)\varepsilon^n, \forall n \in \mathbb{N}$.

Case $c^n \neq 1\, \forall n \in \mathbb{N}, \lambda = 1$

Proposition

Let $\varphi(z) = cz, z \in \mathbb{D}, |c| = 1$ and c is not a root of unity. Then, the following are equivalent:

- $1 \in \rho(C_\varphi, A_0^{-\infty})$,
- $\exists\, s \geq 1$ and $\varepsilon > 0$ such that $|c^n - 1| \geq \varepsilon n^{-s}$ for each $n \in \mathbb{N}$,
- $c = e^{i2\pi x}$, where x is a Diophantine number.

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Definition

A real number $x \in \mathbb{R}$ is called *Diophantine* if $\exists \delta \geq 1$ and $d(x) > 0$ such that

$$\left| x - \frac{p}{q} \right| \geq \frac{d(x)}{q^{1+\delta}}$$

for all rational numbers p/q.

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